

## REVERSE CHAIN RULE TYPE INTEGRATION

Recall if  $y = f(g)$ , then  $y' = f' \cdot g'$ . So  $\int (f' \cdot g') dx = f(g) + c$ .

Useful method: - write down a function which could be a likely answer  
(it is usually the bracket term, but increase the index by 1)  
- differentiate to work out if you need to adjust a constant.

Example: Integrate

1)  $\int 6x(x^2 + 5)^3 dx$

$$\frac{d}{dx} (x^2 + 5)^4 = 4 \cdot (x^2 + 5)^3 \cdot 2x = 8x(x^2 + 5)^3 \rightarrow \int 8x(x^2 + 5)^3 dx = (x^2 + 5)^4 + c$$

$$\therefore \int 6x(x^2 + 5)^3 dx = \frac{6}{8} \int 8x(x^2 + 5)^3 dx = \frac{3}{4} (x^2 + 5)^4 + c$$

2)  $\int 4xe^{x^2+1} dx$

$$\frac{d}{dx} e^{x^2+1} = 2x \cdot e^{x^2+1} \rightarrow \int 2xe^{x^2+1} dx = e^{x^2+1} + c$$

$$\therefore \int 4xe^{x^2+1} dx = 2 \int 2xe^{x^2+1} dx = 2e^{x^2+1} + c$$

3)  $\int (3x + 1) \cos(3x^2 + 2x) dx$

$$\frac{d}{dx} \sin(3x^2 + 2x) = (6x + 2) \cos(3x^2 + 2x) = 2(3x + 1) \cos(3x^2 + 2x)$$

$$\rightarrow \int 2(3x + 1) \cos(3x^2 + 2x) dx = \sin(3x^2 + 2x) + c$$

$$\therefore \int (3x + 1) \cos(3x^2 + 2x) dx = \frac{1}{2} \int 2(3x + 1) \cos(3x^2 + 2x) dx = \frac{1}{2} \sin(3x^2 + 2x) + c$$

4)  $\int \sqrt{y-1} dy$

$$\frac{d}{dx} (y-1)^{3/2} = \frac{3}{2} (y-1)^{1/2} = \frac{3}{2} \sqrt{y-1} \rightarrow \int \frac{3}{2} \sqrt{y-1} dx = (y-1)^{3/2} + c$$

$$\therefore \int \sqrt{y-1} dx = \frac{2}{3} \int \frac{3}{2} \sqrt{y-1} dx = \frac{2}{3} (y-1)^{3/2} + c$$

Formulaic method:

$f(x)$ is linear, e.g. $f(x) = ax + b$	$f(x)$ is non-linear, e.g. $f(x) = ax^n + bx + c$
Rule: $\int [f(x)]^n dx$	Rule: $\int f'(x) \cdot [f(x)]^n dx$
$= \frac{1}{(n+1)[f'(x)]} \cdot [f(x)]^{n+1} + c$	$= \frac{1}{(n+1)} \cdot [f(x)]^{n+1} + c$
Example:	Example:
$\int \sqrt{x+1} dx$ $= \int (x+1)^{1/2} dx$ $= \frac{1}{(\frac{1}{2}+1) \cdot 1} \cdot (x+1)^{\frac{1}{2}+1} + c$ $= \frac{1}{3/2} \cdot (x+1)^{3/2}$ $= \frac{2}{3} \cdot (x+1)^{3/2} + c$	$\int 6x(x^2+5)^3 dx$ $= \int 3 \cdot 2x(x^2+5)^3 dx$ $= 3 \cdot \frac{1}{3+1} \cdot (x^2+5)^{3+1} + c$ $= \frac{3}{4} (x^2+5)^4 + c$ <div style="text-align: right;"> <math>f(x) = (x^2+5)</math>  <math>f'(x) = 2x</math> </div>

### SPECIAL CASES:

1) Trig functions:  $\int f'(x) \cdot \cos[f(x)] dx = \sin[f(x)] + c$

$$f(x) = (3x^2 + 2x)$$

$$f'(x) = 6x + 2$$

$$\text{Example: } \int (3x+1) \cos(3x^2+2x) dx = \int \frac{1}{2} (6x+2) \cos(3x^2+2x) dx$$

$$= \frac{1}{2} \sin(3x^2+2x) + c$$

$$= \frac{1}{2} \sin(3x^2+2x) + c$$

2) Exponential functions:  $\int f'(x) \cdot e^{f(x)} dx = e^{f(x)} + c$

$$\text{Example: } \int 4xe^{x^2+1} dx = \int 2 \cdot 2x \cdot e^{x^2+1} dx$$

$$= 2 \cdot e^{x^2+1} + c$$

$f(x) = (x^2+1)$   
 $f'(x) = 2x$